

Study of aerosol time series data using Tsallis statistics and fractal analyzes

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Abstract: We consider herein the stationary of the distributions for time series of aerosol particles concentration using Tsallis statistics and some elements of fractal analyzes. Using accumulation mode records data series taken in an Albanian small observatory station, we find that 1 minute time records seems to be in a highly instable regime which tends to be relaxed when data are grouped on segments equivalents to 5-10 minutes or more. Fractal structure is very noisy or undetectable on the very short recording time data series and become apparent when we regroup data on 10 minutes equivalent or more.

Keywords: aerosol distributions, complex systems, Tsallis statistics.

1. Introduction

The size distribution of atmospheric aerosols, together with their composition, sources, is a key element in understanding and managing aerosol effects on health, visibility, and climate [1]. Therefore the multidimensional study of such systems will bring important information relevant to its increased overall impact and for its complex system particularly. The studies and researches for aerosols and other air particles usually consider the distribution according to the particles diameter. Time series evolution of aerosols particles concentration is considered in framework of complex phenomena underlying size evolution in particular area [2]. There are many papers reporting static and dynamic distribution as in [3]. Here we focus to a statistical point of view looking for the stationary analyze on distributions. We use data recorded in Albania by Aerosol Spectrometer GRIMM model 109, for a large specter of particle size [9]. Measures are taken in differential and accumulation mode. In the differential mode the number of particles within a range centered on the channel values is recorded, whereas in the accumulation mode all particles with mobility radius grater than the value assigned on the channel index are recorded. According to the Tsallis statistics, the MaxEnt principle using variance constraints will lad to a q-Gaussian distribution [4][5]. Here the parameter q will report the distance form the stationary state [4]. Considering the system encompassing all processes and particles, we propose to extract particular information on the complexity, fractal structure and possible self organization behaviour by the analysis of q-distributions, the Tsallis triplet. Focusing our attention on our home data records as specific system, we use the other open source data to comparison purposes. We mention here that q-exponentials comes from a deformed algebra and are defined as $e_q(x) = \max((1 + (1 - q)x)^{\frac{1}{1-q}}, 0)$, providing that the base of the power is positive. When $q \rightarrow 1$ the familiar exponential function is recovered. It has been

established that physical systems characterized by long-range interactions or long-term memory or being of a multi-fractal nature are best described by a generalized statistical-mechanics formalism proposed by Tsallis [7]. We will adopt this method for the study of the system of aerosols particles and their interaction.

2. The q-statistics

When N-particle system reaches the thermodynamic equilibrium, the standard statistical theory identifies the distribution according to a parameter as consequence of the entropy maximization. Hence, the optimization of the classical Boltzman-Gibs entropy $S = -\sum_{i=1}^w p_i \ln p_i$ imposing constraints of the non divergence for the mean and standard deviation, leads to the canonical exponential $p_i \sim e^{-\beta(x-\mu)}$ and Gaussian-like distribution $p_i \sim e^{-\beta(x-\mu)^2}$, by standard techniques of statistical mechanics. These assumptions fail to explain ‘fat tail’ distributions that are common for a large number of complex systems. C.Tsallis proposed to write the entropy of the system as $S_q^T = \frac{1}{q-1} \left[1 - \sum_{i=1}^{\Omega} p_i^q \right]$ in the discrete case which in the continuous case leads to the q-exponential distribution $p(x) = A \{1 - \beta(1 - q)(x - \mu)\}^{\frac{1}{1-q}}$ if one makes use of the mean constraint, and the q-Gaussian distribution which reads $p(x) = A \left\{ 1 - \beta(1 - q)[x - \bar{x}]^2 \right\}^{\frac{1}{1-q}}$ [4],[7] if variance is a constraint too. Here the parameter q is considered as the distance of the system from the equilibrium state, or more precise, from the stationary state. Next we evaluate the sensitivity q-parameter according to the formula

$\frac{1}{q_{sensitive}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}}$ where $\alpha_{min,max}$ are the singularity point of the fractal power function of the structure [4][5]. Finally we estimate the last parameter q that stand for relaxation rate and is found from the q-exponential fitted to the time correlation function of the series.

3. Q-distributions for aerosol concentrations

The first series of data considered here consist of the values of concentration of the particles with diameter size in the range 0.25 nm to 5000 nm, taken every minute and covering the 2 month period February-Mars at the 2011, measured at a small station situated in Tirana [6]. The discrete distributions were obtained by standard optimization of the histograms according to the Scot rule. Fitting distributions for concentration found on each channel, we see that for series found from first channels (small mobility radius) the q-Gaussian is typically characteristic in the range [280nm-400nm], and it becomes sharper in the range [800nm-2000nm], degenerating to a power

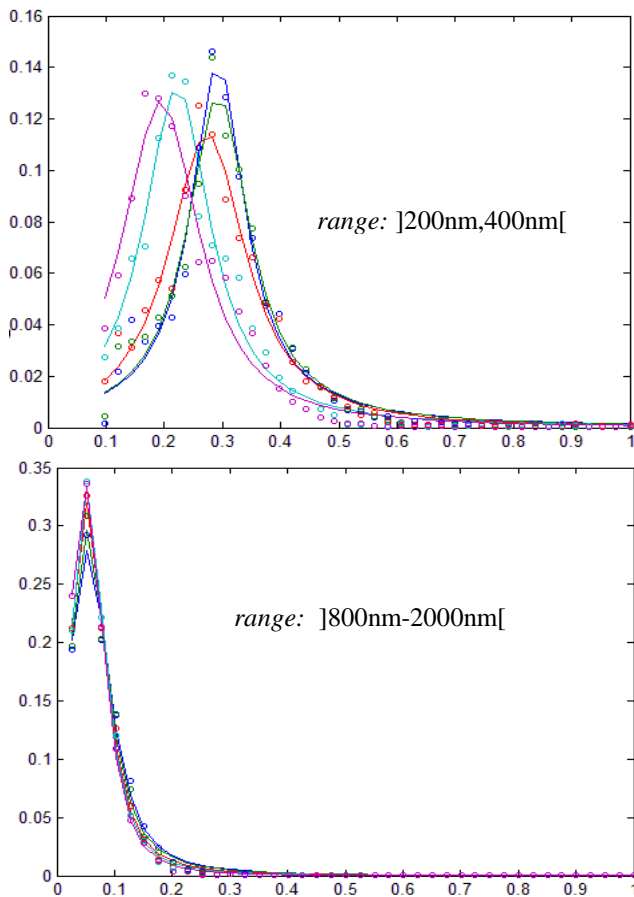


Figure 1: q-distributions for concentration of aerosols (different mobility radius)

law-like function for bigger size (Fig1). The stationary parameter is found $q \sim [1.8-1.6]$ with an error of 1.5%. Its value drops smoothly with the increase of the aerosols size. The same is observed for the relaxation parameter where $q \sim [5.9-5.2]$. The sensitive q-parameter is not known as the fractal structure is not clear. In the particle size of [800nm-2000nm] we found the stationary parameter $q \sim [2.1-1.9]$ and again its value drops as the size increases. The other two q-Tsallis parameters are not determinable. To this point we conclude that the system is far from the stationary state and it becomes more unstable for

higher value of registration channels in accumulation mode. Here we must take into account the registration time, because the interval of the records could affect or at least could perturb the stability of the measurements. In a second consideration we brought together the number of particles found on n successive measures that is if records were taken on n minutes intervals. Scanning those new time lags, we stop if fractal structure is restored, so we can calculate the sensitivity to the initial condition meaning a dynamic view is possible. Considering accumulation time of 10 minutes, we obtain a clear picture of the fractal regime (Fig2) in the range [800nm,3000nm]. Here we find that the sensitivity parameter grows in line of increasing value of channel filters, beginning from negative value for small filter threshold. Q-Gaussian fits well with distributions, but again, they become noisy as the channel threshold increases. For 11 channels on the range of radius [900nm;1500nm] the value of q-stationary parameter are found to be [1.46 1.47 1.56 1.49 1.52 1.52 1.77 1.18 1.07 1.53 1.52] with an error less to 2.5%. The relaxation shows strong toss and therefore we do not report values. Nevertheless, the fractal structures seems to be well stabilized, and the values of q-parameters are in compliance

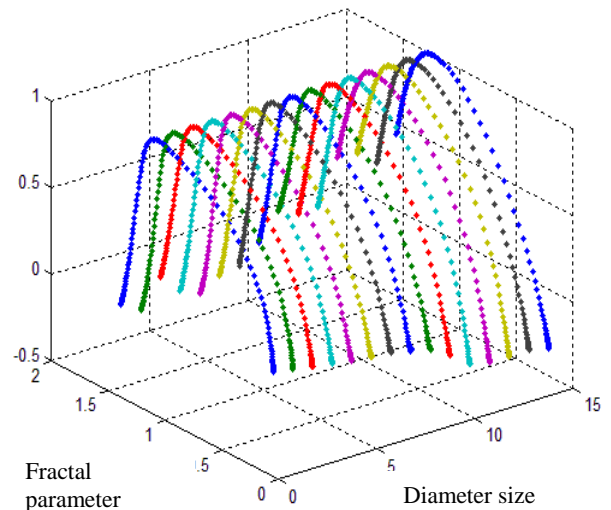


Figure 3: Fractal spectrum, range [250nm-900 nm]. Cumulative time 10 minutes

with Tsallis formalism, $q_{sensitive} < q_{stationary}$ [4],[5] as $q_{sensitive} = [-0.23 -0.153 -0.11 -0.06 -0.085 0.008 0.011 0.034 0.056 0.114 0.195]$. Taking $n=30$ that is a cumulative time of 30 minutes, all the three q-parameters are well determined covering the full range. Accordingly, in the radius range [900nm-1500nm] the results are as follow:

r(nm)	800	1000	1300	1600	2000	2500
q_{sens}	0.1277	0.1549	0.1906	0.2481	0.2670	0.2949
q_{stac}	1.5413	1.4144	1.4013	1.3289	1.0001	1.0000
q_{relax}	2.8670	2.8590	2.8546	2.8626	2.8661	2.8941
R(nm)	3000	3500	4000	5000	6500	7500
q_{sens}	0.3292	0.3622	0.3907	0.4038	0.4414	0.4715
q_{stac}	1.0016	1.0000	1.0000	1.2088	1.1862	1.5619
q_{relax}	2.9090	2.9030	2.8347	2.7727	2.7757	2.7538

Reading those results, and following general rules $q_{sens} \leq q_{stat} \leq q_{relax}$, we do not report stable regime if $q_{stationary}=1$, since if the real stable point is reached, all three

q-Tsallis parameters must be 1. For example, for $r > 2500\text{nm}$ we obtain $q_{stat} = 1.0009 \mp 5e-5$ but $(q_{sens} = 0.2949 \pm 0.023) \ll 1$ and $(q_{rel} = 2.8941 \pm 0.157) \gg 1$, so it is not a stable regime. In the light of q-distributions we identify a fractal structure present on the system, indicating the presence of a more organized phase (Fig3). It could be related to some interior timing parameters on the system, or cooperative effect of the measurement and interior evolution that produce and destroy particles affecting their number that reach the recording equipment. We compared those results with other data series taken from open sources (a Milan Observatory) whereof we see that the pictures are similar for different accumulation time respectively. Hence qualitative findings for accumulation time of 5 minutes in our observatory behave like the ones obtained of 30 minutes interval for Milan reference data. We conclude that important information is revealed or obscured according to the distance of system's state from the stationary state, and in this point Tsallis distributions analyses will become very useful. Other elements as the ion-aerosol distribution evolution will be considered on our incoming works.

4. Conclusions

The systems of aerosols and of other air floating particle shows specific behaviour that can be described using standard terms used for the analyses of the complex systems. Generally they are far from stationary state due to the complex phenomena underlying their interaction or their sources. If measurement on such systems is carefully taken according to the accumulation time particularly, fractal and other self organizing behaviour do emerge. In this case, it is possible to identify elements of complexity that otherwise are not visible. This will help researchers to better accommodate methods of measurement and evaluation of particular values, as usually they are strongly affected from their non-stationary distribution resulting from time recording. Moreover, such analysis could contribute in the knowledge of interior time of interaction or

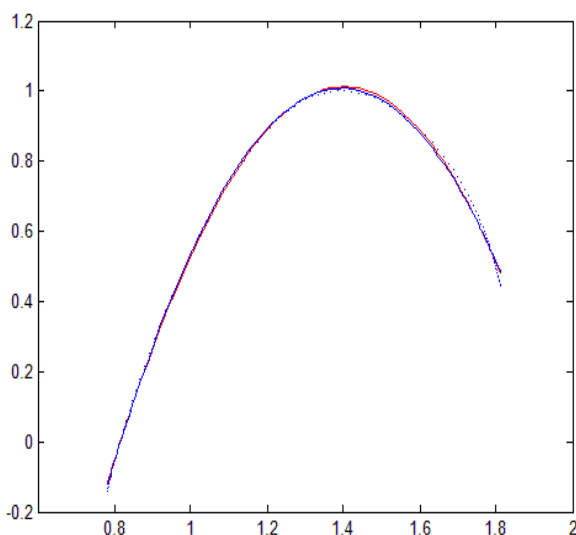


Figure 3. Fractal spectrum. Channel $r > 0.25\mu\text{m}$, accumulation modes, cumulative time 5 minutes

relaxation activities, related to the very complex phenomena of these particles growth and their lifetime.

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